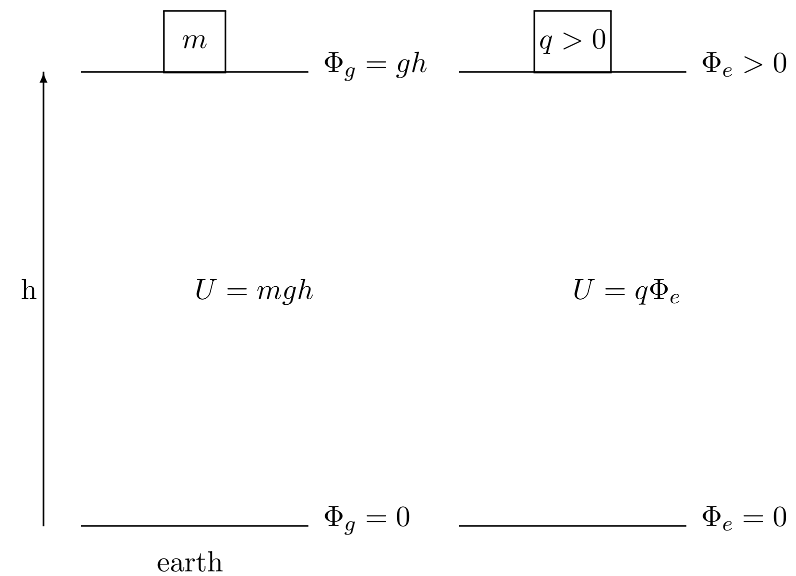
1 Gravitational and Electrostatic Potential Energy

The electrostatic and gravitational forces are two of the four fundamental forces of nature. The electrosatic force dictates the motions of electrons and nuclei in atoms and molecules and, hence, is responsible for the matter around us. The much weaker graviational force governs the motion of large objects and is responsible for the structure of the earth, the solar system and beyond. The correspondence between gravitational and electrostatic potential energy can be illuminated with the diagram below.



Gravitational Potential Energy Electrostatic Potential Energy

When a mass m is raised a short distance in the gravitational field at the surface of the earth, the change in gravitational potential energy is $U = mgh$, where m is the mass in kg, $g = 9.8 m/s^2$ is the acceleration due to gravity at the surface of the earth in m/s^2 and h is the height in meters. If the mass is released from the height h, it will experience a downward force $F = mg$, and when it hits the ground all of the initial potential energy will have been converted to kinetic energy.

For the electrostatic case, a charge q can be moved between two plates separated by the distance h, between which there is a difference in the electrostatic potential Φ_e measured in Volts. When a charge q is moved through a change in the electrostatic potential Φ_e , the change in potential energy is $U = q\Phi_e$, where q is the charge in Coulombs and Φ_e is the electrostatic potential in Volts. If q is positive and the potential change going from the bottom plate to the top plate is also positive, then when the charge is release it will experience a downward force $F = q\Phi_e/h$, where Φ_e/h is the electric field in the downward direction. When it hits the bottom plate all of the initial potential energy will have been converted to kinetic energy.

The same correspondence can be made for the gravitational potential for a point source

$$\Phi_g(r) = -\frac{GM}{r} \tag{1}$$

and for the electrostatic potential for a point source

$$\Phi_e(r) = \frac{Q}{4\pi\epsilon_o r}. \tag{2}$$

In these expressions, r is the radial distance from the point source, the gravitational constant $G = 6.67 \times 10^{-11} Nm^2/kg^2$, M is the mass of the point source in kg, Q is the charge of the point source in Coulombs and the permittivity of free space is $\epsilon_o = 8.85 \times 10^{-12} C^2/Nm^2$. When an object is positioned at a radial distance r from a point source, the gravitational potential energy relative to that when the object is infinitely far away is

$$U(r) = m\Phi_g = -\frac{GMm}{r}, \tag{3}$$

and the electrostatic potential energy is

$$U(r) = q\Phi_e = \frac{qQ}{4\pi\epsilon_o r}. \tag{4}$$

The change in gravitational potential energy when a mass m is moved from r_1 to r_2 is

$$\triangle U = -GMm(\frac{1}{r_2} - \frac{1}{r_1}). \tag{5}$$

The change in electrostatic potential energy when a charge q is moved from r_1 to r_2 is

$$\triangle U = \frac{Qq}{4\pi\epsilon_o}(\frac{1}{r_2} - \frac{1}{r_1}). \tag{6}$$

For both point-source potentials, the force on an object m or q can be expressed as the negative gradient of the potential energy

$$\vec{\mathbf{F}} = -\vec{\nabla} U(r). \tag{7}$$

For the gravitational case,

$$\vec{\mathbf{F}} = -\frac{GMm}{r^2}\hat{r}, \tag{8}$$

and for the electrostatic case

$$\vec{\mathbf{F}} = \frac{Qq}{4\pi\epsilon_o r^2}\hat{r}, \tag{9}$$

where $\hat{r}$ is a unit vector in the outward radial direction.